LECTURE 6

MODELING EARTHQUAKES AS TSUNAMI SOURCES

PRINCIPLES of HYDRODYNAMIC SIMULATIONS

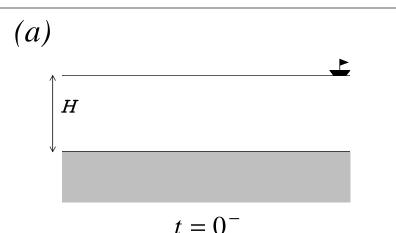
CLASSICAL APPROACH

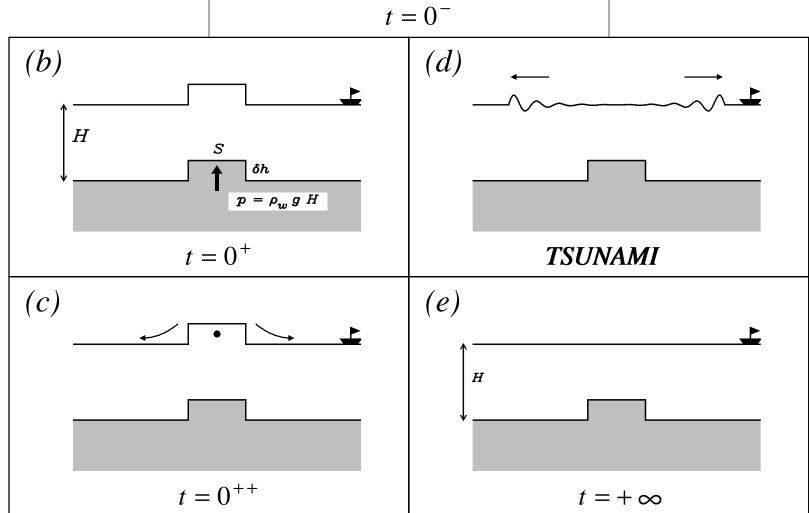
- 1. Obtain model of Earthquake Rupture
- 2. Compute Static Deformation of Ocean Bottom
- 3. Use as Initial Conditions of Vertical Surface Displacement with Zero Initial Velocity
- **4.** Run Hydrodynamic Model (*e.g.*, **MOST**)
- 5. Propagate, up to and including

 INUNDATION of Receiving Shore

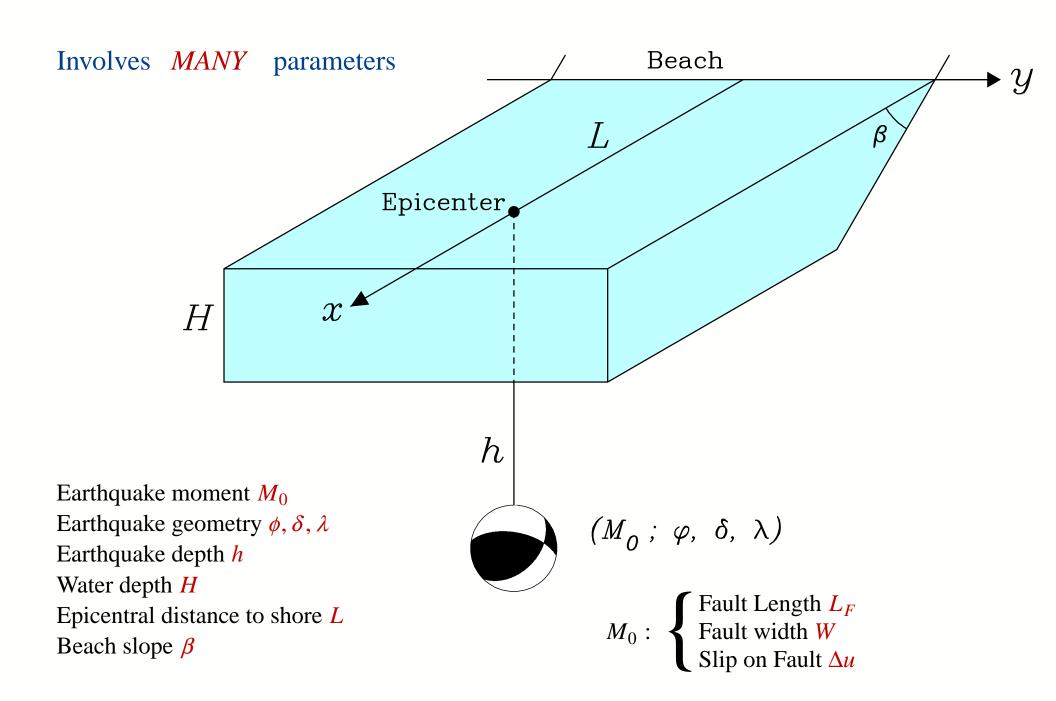
CLASSICAL

APPROACH



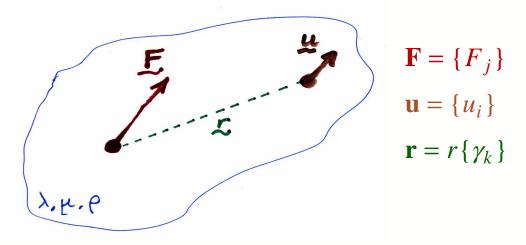


GENERIC EARTHQUAKE DISLOCATION



FIRST STEP

Position a point force F in an infinite homogeneous elastic medium



→ Obtain the *Dynamic* displacement field of the deformation

Then for a point force $X_0(t)$ in the x_i -direction at the origin, we have

$$u_{i}(\mathbf{x}, t) = X_{0} * G_{ij} \quad \text{(in the notation of Chapter 3)}$$

$$= \frac{1}{4\pi\rho} (3\gamma_{i}\gamma_{j} - \delta_{ij}) \frac{1}{r^{3}} \int_{r/\alpha}^{r/\beta} \tau X_{0}(t - \tau) d\tau$$

$$+ \frac{1}{4\pi\rho\alpha^{2}} \gamma_{i}\gamma_{j} \frac{1}{r} X_{0} \left(t - \frac{r}{\alpha}\right)$$

$$- \frac{1}{4\pi\rho\beta^{2}} (\gamma_{i}\gamma_{j} - \delta_{ij}) \frac{1}{r} X_{0} \left(t - \frac{r}{\beta}\right). \tag{4.23}$$

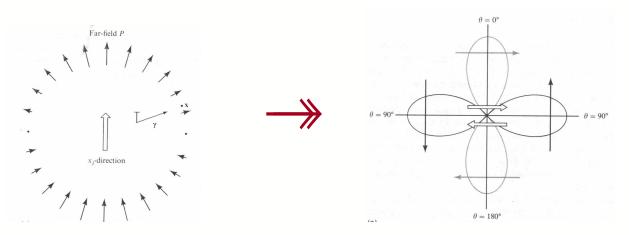
[Aki and Richards, 1980; p. 73, Eqn. (4.23)]

• The *STATIC* displacement is simply obtained by putting $t \to \infty$.

[This expression is known as the Somigliana Tensor]

SECOND STEP

Replace Single Force by Double-Couple



→ Simply use Somigliana's tensor as a Green's function and take appropriate derivatives.

→ Note that these are the P and S waves of the near [and far] field[s]. NEAR FIELD NEAR FIELD

[Far Field]

$$M_{pq} * G_{np,q} = \left(\frac{15\gamma_{n}\gamma_{p}\gamma_{q} - 3\gamma_{n}\delta_{pq} - 3\gamma_{p}\delta_{nq} - 3\gamma_{q}\delta_{np}}{4\pi\rho}\right) \frac{1}{r^{4}} \int_{r/\alpha}^{r/\beta} \tau M_{pq}(t-\tau) d\tau$$

$$+ \left(\frac{6\gamma_{n}\gamma_{p}\gamma_{q} - \gamma_{n}\delta_{pq} - \gamma_{p}\delta_{nq} - \gamma_{q}\delta_{np}}{4\pi\rho\alpha^{2}}\right) \frac{1}{r^{2}} M_{pq} \left(t - \frac{r}{\alpha}\right)$$

$$- \left(\frac{6\gamma_{n}\gamma_{p}\gamma_{q} - \gamma_{n}\delta_{pq} - \gamma_{p}\delta_{nq} - 2\gamma_{q}\delta_{np}}{4\pi\rho\beta^{2}}\right) \frac{1}{r^{2}} M_{pq} \left(t - \frac{r}{\beta}\right)$$

$$+ \frac{\gamma_{n}\gamma_{p}\gamma_{q}}{4\pi\rho\alpha^{3}} \frac{1}{r} \dot{M}_{pq} \left(t - \frac{r}{\alpha}\right)$$

$$- \left(\frac{\gamma_{n}\gamma_{p} - \delta_{np}}{4\pi\rho\beta^{3}}\right) \gamma_{q} \frac{1}{r} \dot{M}_{pq} \left(t - \frac{r}{\beta}\right). \tag{4.29}$$

[Aki and Richards, 1980; p. 79; Eqn. (4.29)]

THIRD STEP

• Include effect of free surface

(Combine with "reflection" of equivalent P and S waves)

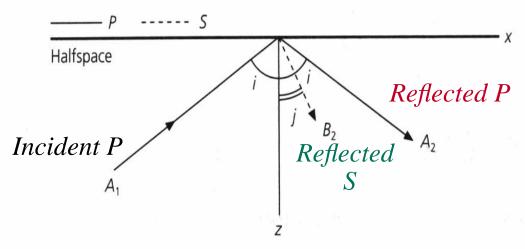


Fig. 2.6-6 Geometry for a P wave in a halfspace incident upon a free surface. A_1 , A_2 , and B_2 are the amplitudes of the incident P, reflected P, and reflected SV waves.

[Stein and Wysession, 2002]

Integrate over finite area of faulting

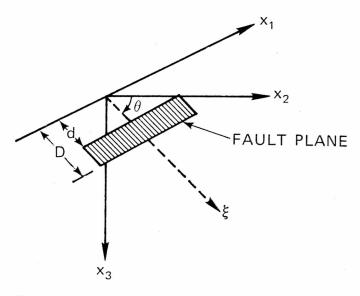


Fig. 1. Fault geometry and coordinate system.

The problem has an analytical solution

TWO equivalent algorithms

Mansinha and Smylie [1971]

Okada [1985]

Only difference: Okada allows for tensile crack (non-double-couple solution).

STATIC DEFORMATION OF OCEAN BOTTOM

Straightforward, if somewhat arcane analytical formulæ

[Mansinha and Smylie, 1971; Okada, 1985]

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YOSHIMITSU OKADA

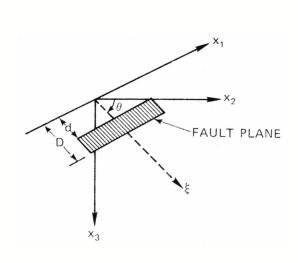
(1) Displacements

For strike-slip

$$\begin{cases} u_x = -\frac{U_1}{2\pi} \left[\frac{\xi q}{R(R+\eta)} + \tan^{-1} \frac{\xi \eta}{qR} + I_1 \sin \delta \right] \\ u_y = -\frac{U_1}{2\pi} \left[\frac{\tilde{y}q}{R(R+\eta)} + \frac{q \cos \delta}{R+\eta} + I_2 \sin \delta \right] \\ u_z = -\frac{U_1}{2\pi} \left[\frac{\tilde{d}q}{R(R+\eta)} + \frac{q \sin \delta}{R+\eta} + I_4 \sin \delta \right] \\ \end{bmatrix}.$$

For dip-slip

$$\begin{cases} u_x = -\frac{U_2}{2\pi} \left[\frac{q}{R} - I_3 \sin \delta \cos \delta \right] \\ \\ u_y = -\frac{U_2}{2\pi} \left[\frac{\tilde{y}q}{R(R+\xi)} + \cos \delta \tan^{-1} \frac{\xi \eta}{qR} - I_1 \sin \delta \cos \delta \right] \\ \\ u_z = -\frac{U_2}{2\pi} \left[\frac{\tilde{d}q}{R(R+\xi)} + \sin \delta \tan^{-1} \frac{\xi \eta}{qR} - I_5 \sin \delta \cos \delta \right] \\ \\ \end{bmatrix} \right].$$



where

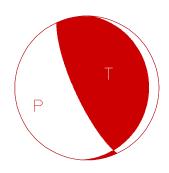
$$\begin{cases} I_1 = \frac{\mu}{\lambda + \mu} \left[\frac{-1}{\cos \delta} \frac{\xi}{R + \tilde{d}} \right] - \frac{\sin \delta}{\cos \delta} I_5 \\ I_2 = \frac{\mu}{\lambda + \mu} \left[-\ln(R + \eta) \right] - I_3 \\ I_3 = \frac{\mu}{\lambda + \mu} \left[\frac{1}{\cos \delta} \frac{\tilde{y}}{R + \tilde{d}} - \ln(R + \eta) \right] + \frac{\sin \delta}{\cos \delta} I_4 \\ I_4 = \frac{\mu}{\lambda + \mu} \frac{1}{\cos \delta} \left[\ln(R + \tilde{d}) - \sin \delta \ln(R + \eta) \right] \\ I_5 = \frac{\mu}{\lambda + \mu} \frac{2}{\cos \delta} \tan^{-1} \frac{\eta(X + q \cos \delta) + X(R + X)\sin \delta}{\xi(R + X)\cos \delta} \end{cases}$$

STATIC DEFORMATION OF OCEAN BOTTOM

EXAMPLE: VALPARAISO, CHILE

17 AUGUST 1906

$$M_0 = 2.8 \times 10^{28} \text{ dyn-cm}$$

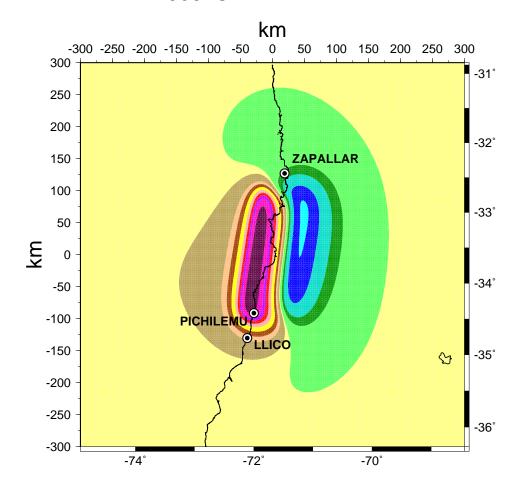


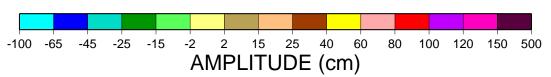
$$\phi_f = 3^{\circ}; \delta = 15^{\circ}; \lambda = 117^{\circ}$$

$$L_F = 200 \text{ km}; W = 75 \text{km};$$

 $\Delta u = 5.3 \text{ m}$

1906 CHILEAN EVENT





[Okal, 2005]

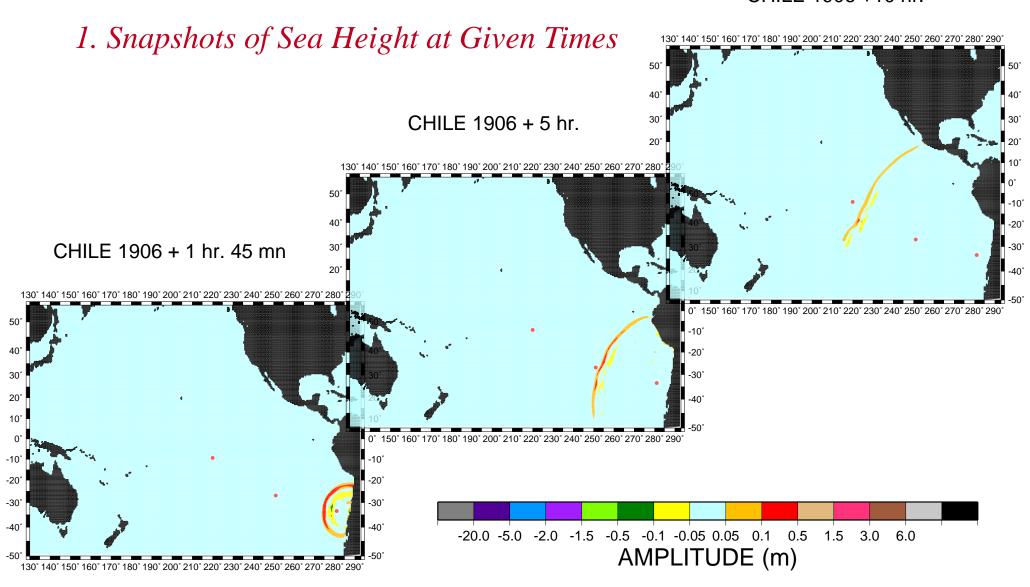
• Use this static deformation field (limited to its oceanic portion) as the initial condition $(t = 0_+)$ of the hydrodynamic calculation.

→ Justification: The seismic source is generally MUCH FASTER than any tsunami process, hence it can be taken as instantaneous.

(even in the case of SLOW, so-called "Tsunami" earhtquakes)

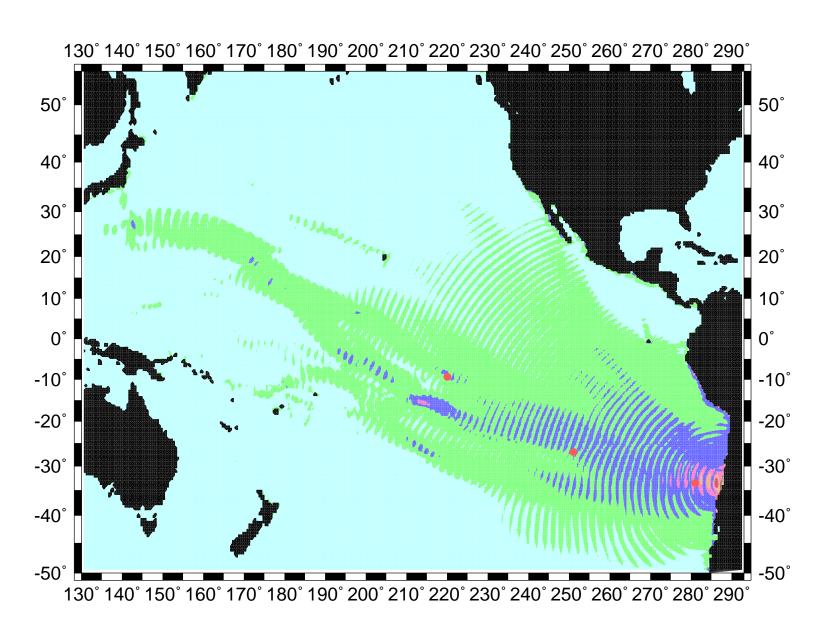
PRODUCTS OF SIMULATION

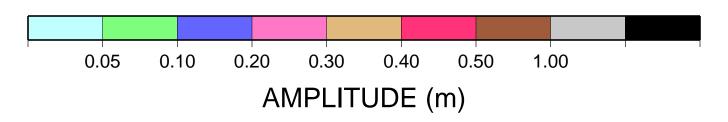
CHILE 1906 +10 hr.



PRODUCTS OF SIMULATION

2. Map of Maximum Amplitude of Tsunami Wave





HOW ROBUST IS THIS PROCEDURE?

It is worth exploring the robustness of our results in the far field, with respect to detailed parameters of our sources, *a fortiori* unknown in the context of many simulations.

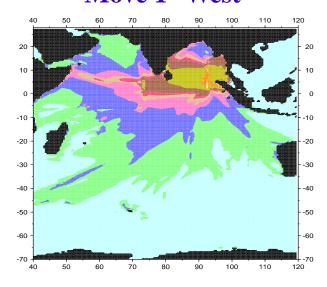
We study simulated amplitudes of the 2004 Sumatra-Andaman tsunami in the far field under fluctuations of source parameters, while keeping the seismic moment of the source constant.

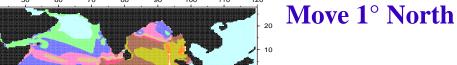
We conclude that our results are indeed robust.

The primary parameters controlling the far field tsunami amplitudes are the size (moment) of the parent earthquake and the depth of the water column in the epicentral area.

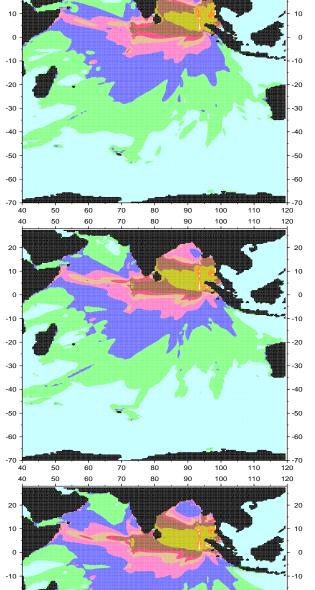
1. MOVE SOURCE LATERALLY

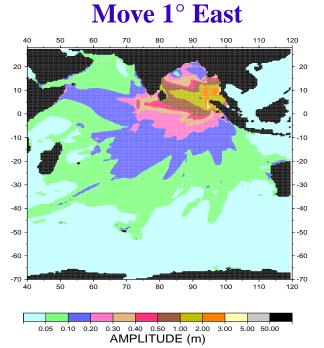
Move 1° West



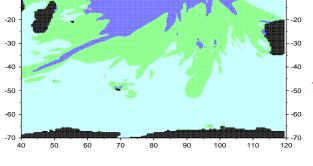


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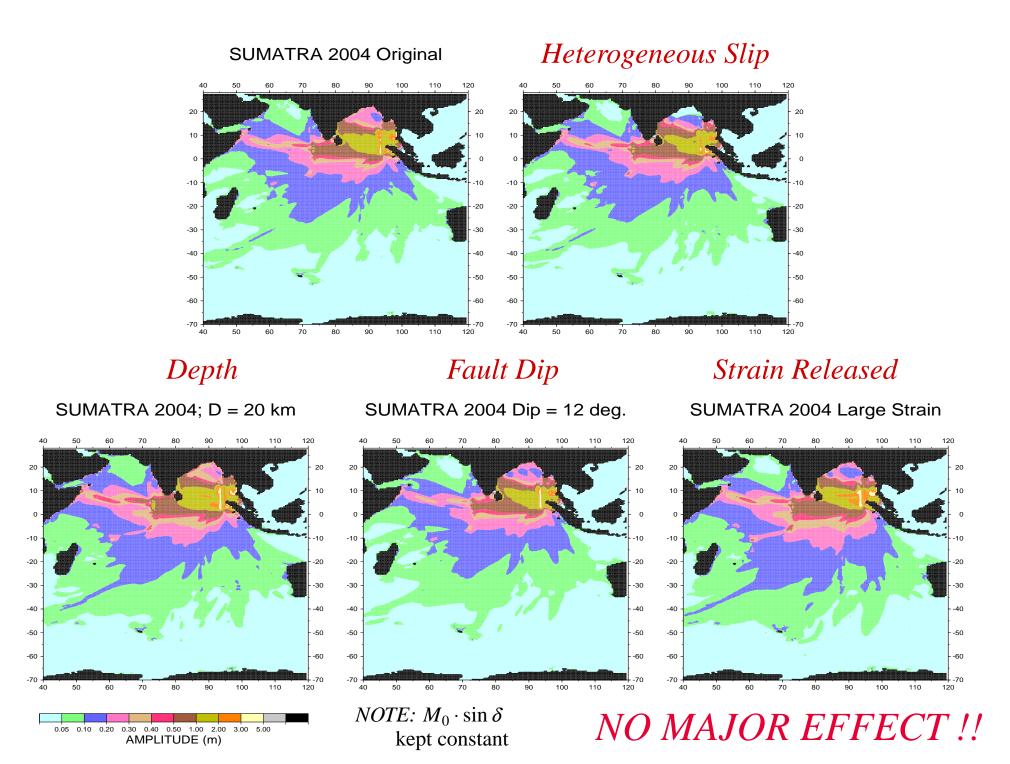


Move 1° South -60



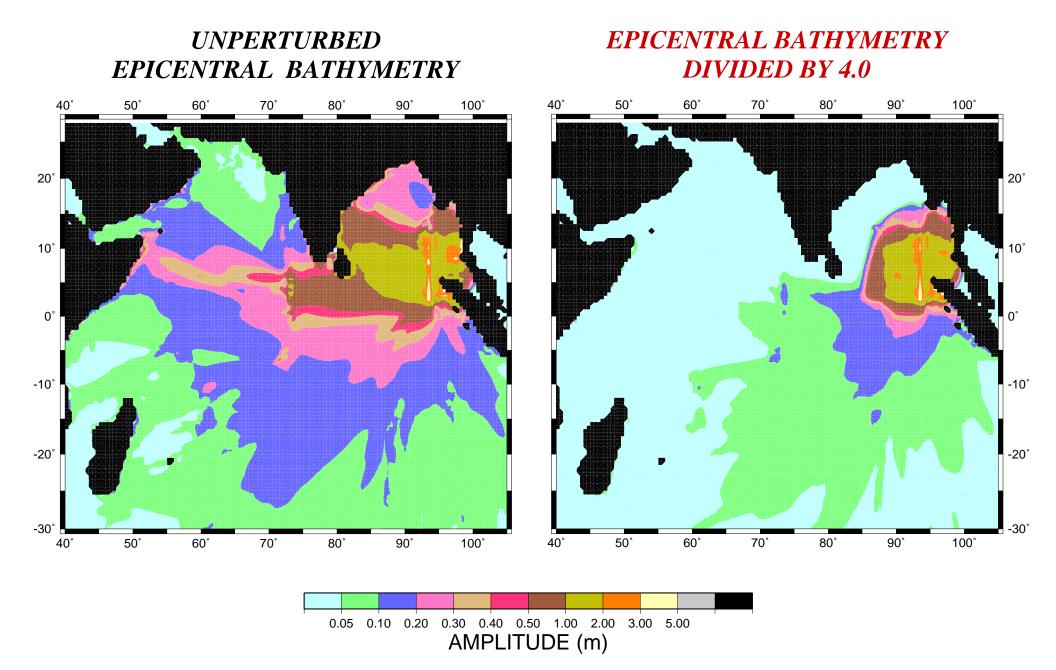
NO MAJOR EFFECT!!

2. CHANGE SOURCE PARAMETERS



By CONTRAST, WATER DEPTH at the SOURCE PLAYS a CRUCIAL ROLE

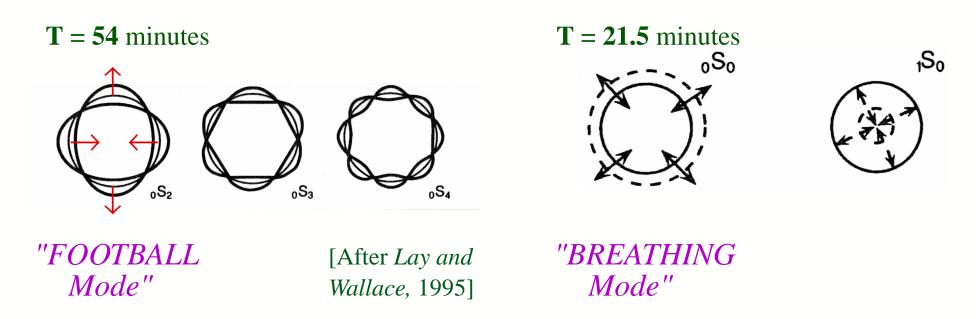
NOTE: This explains the much smaller tsunami during the 2005 Nias earthquake.



NORMAL MODE FORMALISM: A different approach

[Ward, 1980]

- At very long periods (typically 15 to 54 minutes), the Earth, because of its finite size, can ring like a bell.
- Such *FREE OSCILLATIONS* are equivalent to the superposition of two progressive waves travelling in opposite directions along the surface of the Earth.



Ward [1980] has shown that **Tsunamis come naturally as a special branch of the normal modes of the Earth,** provided it is bounded by an ocean, and gravity is included in the formulation of its vibrations.

In the normal mode formalism, the solution of the vertical displacement (both in the water and solid Earth) is sought as

$$u_z(\mathbf{x};t) = u_z(r,\theta,\phi;t) = y_1(r) \cdot Y_l^m(\theta,\phi) \exp(i\,\omega\,t) = y_1(r) \cdot P_l^m(\theta,\phi) \cdot e^{i\,m\,\phi} \cdot \exp(i\,\omega\,t)$$

where Y_l^m is a *spherical harmonic* of order l and degree m; P_l^m is the Legendre polynomial of order l and degree m; and $\{r, \theta, \phi\}$ is a system of spherical polar coordinates.

This allows for the **separation** of the variables $\{r, \theta, \phi\}$.

The problem is complemented by similar expressions for the overpressure $p = -y_2$ in the tsunami wave, the horizontal displacement $u_x = l \cdot y_3$, and the change in the gravity potential y_5 .

Under the linear approximation, the equations of hydrodynamics transform into a system of linear differential equations of the first order.

For any given l, i.e., wavenumber k = (l + 1/2) (a radius of the Earth), the system has non trivial solutions for only one value of ω . The relationship between l and ω is the *Disppersion Relation of the Tsunami*.

SPHEROIDAL MODE HAS 6-COMPONENT EIGENFUNCTION SATISFYING:

$\frac{dy_1}{dr}$		$\frac{-2\lambda}{(\lambda+2\mu)r}$	$\frac{1}{(\lambda+2\mu)}$	$\frac{L^2 \lambda}{(\lambda + 2\mu) r}$	0	0	0		y_1
$\frac{dy_2}{dr}$		$-\omega^2 \rho + \frac{4\mu(3\lambda + 2\mu)}{(\lambda + 2\mu)r^2} - \frac{4\rho g}{r}$	$\frac{-4\mu}{(\lambda+2\mu)r}$	$L^{2}\left[\frac{\rho g}{r} - \frac{2\mu(3\lambda + 2\mu)}{(\lambda + 2\mu)r^{2}}\right]$	$\frac{L^2}{r}$	0	$-\rho$		<i>y</i> ₂
$\frac{dy_3}{dr}$		$\frac{-1}{r}$	0	$\frac{1}{r}$	$\frac{1}{\mu}$	0	0		<i>y</i> ₃
$\frac{dy_4}{dr}$	=	$\frac{\rho g}{r} - \frac{2\mu(3\lambda + 2\mu)}{(\lambda + 2\mu)r^2}$	$\frac{-\lambda}{(\lambda+2\mu)r}$	$-\omega^{2} \rho + \frac{4\mu L^{2} (\lambda + \mu)}{(\lambda + 2\mu) r^{2}} - \frac{2\mu}{r^{2}}$	$\frac{-3}{r}$	$\frac{-\rho}{r}$	0	•	<i>y</i> ₄
$\frac{dy_5}{dr}$		$4\pi~G~ ho$	0	0	0	0	1		<i>y</i> ₅
$\frac{dy_6}{dr}$		0	0	$\frac{-4\pi L^2 G \rho}{r}$	0	$\frac{L^2}{r^2}$	$\frac{-2}{r}$		<i>y</i> ₆

 y_1 : Vertical displacement y_4 : Tangential stress

 y_3 : Horizontal displacement y_5 : Gravity potential

 y_2 : Normal stress y_6 : Auxiliary gravity

EASILY SOLVED WITH APPROPRIATE BOUNDARY CONDITIONS

EIGENFUNCTIONS of SPHEROIDAL MODES

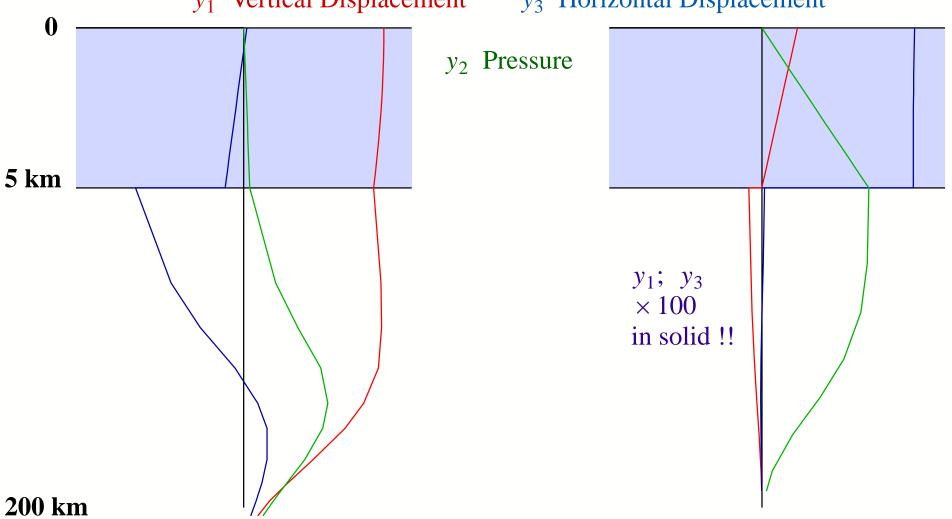
Rayleigh Mode

$$l = 200$$
; $T = 52 s$

Tsunami Mode

$$l = 200$$
; $T = 908 s$

 y_1 Vertical Displacement y_3 Horizontal Displacement



TSUNAMI EIGENFUNCTION is CONTINUED (SMALL) into SOLID EARTH

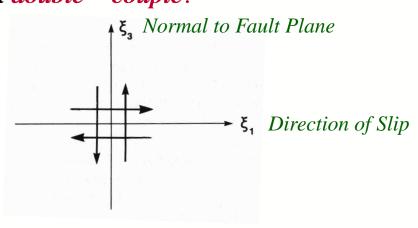
EXCITATION OF TSUNAMI in NORMAL MODE FORMALISM

• Gilbert [1970] has shown that the response of the Earth to a point source consisting of a single force **f** can be expressed as a summation over all of its normal modes

$$\mathbf{u}(r,t) = \sum_{N} \mathbf{s}_{n}(\mathbf{r}) \left(\mathbf{s}_{n}^{*}(\mathbf{r}_{s}) \cdot \mathbf{f}(\mathbf{r}_{s}) \right) \cdot \frac{1 - \cos \omega_{n} t \exp \left(-\omega_{n} t/2 Q_{n} \right)}{\omega_{n}^{2}} ,$$

the *EXCITATION* of each mode being proportional to the *scalar* product of the force \mathbf{f} by the eigen-displacement \mathbf{s} at location $\mathbf{r}_{\mathbf{s}}$.

• Now, an *EARTHQUAKE* is represented by a system of forces called a *double* – *couple*:



The response of the Earth to an earthquake is thus

$$\mathbf{u}(r,t) = \sum_{N} \mathbf{s}_{n}(\mathbf{r}) \left(\varepsilon_{n}^{*}(\mathbf{r}_{s}) : \mathbf{M}(\mathbf{r}_{s}) \right) \cdot \frac{1 - \cos \omega_{n} t \exp \left(-\omega_{n} t/2 Q_{n} \right)}{\omega_{n}^{2}}$$

where the *EXCITATION* is the *scalar product* of the earthquake's **MOMENT** M with the local *eigenstrain* ε at the source $\mathbf{r_s}$.

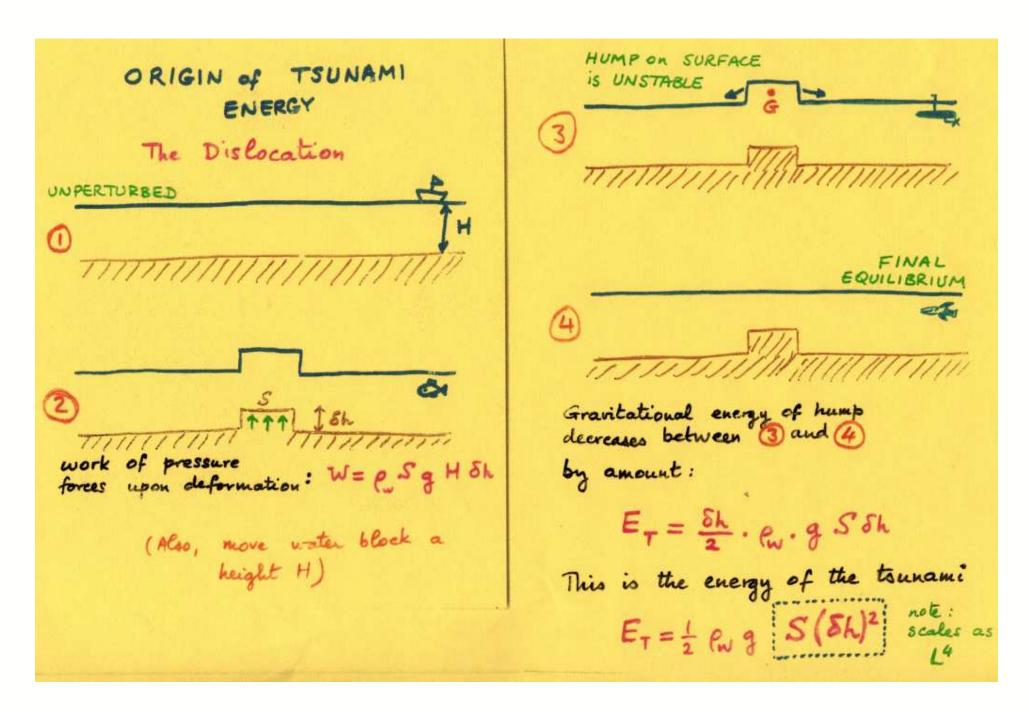
This formula is directly applicable to the case of a tsunami represented by normal modes of the Earth.

ADVANTAGES of NORMAL MODE FORMALISM

- Handles any Ocean-Solid Earth Coupling Including Sedimentary Layers
- Works well at Higher Frequencies
 No need to assume Shallow-Water Approximation

DRAWBACKS of NORMAL MODE FORMALISM

- Must assume Laterally Homogeneous Structure
- Linear Theory -- Does not allow for Large Amplitudes



NOTE: Energy scales as L^4 , i.e., as $M_0^{4/3}$.

ENERGY of a TSUNAMI -- STATIC THEORY [Kajiura, 1981]

$$E = \frac{1}{2} \frac{\rho_w g}{\mu^2} \alpha^{2/3} \cdot F(\delta, \lambda, h, R) \cdot M_0^{4/3} = \frac{1}{2^{4/3}} \frac{\rho_w g}{\mu^{4/3}} \varepsilon_{\text{max}}^{2/3} \cdot F \cdot M_0^{4/3}$$

- * $\alpha = \text{invariant ratio of } M_0 \text{ to } S^{3/2}$
- * *F* : dimensionless factor expressing geometry of faulting, and aspect ratio *R* of fault rupture area.

NOTE: Energy of Tsunami grows faster than Seismic Moment

Energy released by rupture, proportional to M_0 : ε grows like moment.

Hence, Fraction of Earthquake Energy transferred to Tsunami Grows with Earthquake Size

Fortunately, it remains VERY SMALL

(max. 1.3% for Chile, 1960)

TSUNAMI ENERGY COMPUTED from NORMAL MODE THEORY

[Okal, 2003]

• Compute Kinetic Energy of water in Normal Mode Formalism

Note that most energy is carried by HORIZONTAL FLOW

Weigh by excitation function for each mode for given seismic moment M_0 . (averaged over focal geometry)

• Sum over individual modes (equivalent to integrating over frequency)

Account for source spectrum (according to seismic scaling laws)

Account for Finite extent of source depth.

$$E = 0.219 \frac{\rho_w g}{\mu^{4/3}} \cdot \varepsilon_{\text{max}}^{2/3} \cdot M_0^{4/3}$$

Essentially Equivalent to Kajiura's.

E grows as
$$M_0^{4/3}$$

Sumatra 2004:
$$E \approx 7.5 \times 10^{23}$$
 erg (100 times Hiroshima)

WHAT ABOUT THE ATMOSPHERE?

If the tsunami eigenfunction is prolonged into the Solid Earth which is not totally rigid,

• It should be possible to prolong it into the atmosphere, which is not a perfect vacuum.

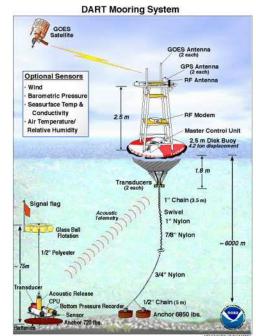
(The sea surface is not a totally "free" boundary)

• This idea, hinted at by Yuen et al. [1970], was proposed by Peltier [1976].

<<<< **STAY TUNED** >>>>>

M_{TSU}

- Use high seas tsunami waveforms recorded by DART system
- Consider tsunami as free oscillation branch of Earth's normal modes [Ward, 1980]
- Recall Magnitude M_m for seismic mantle waves; Define

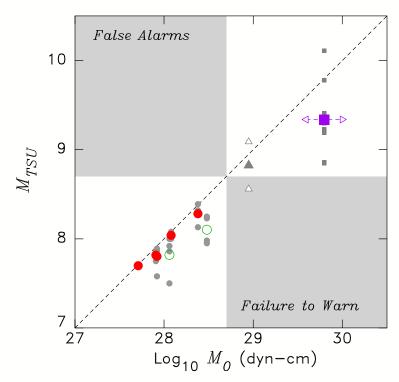


Courtesy of PMEL

$$M_{TSU} = \log_{10} X(\omega) + C_D + C_S + C_0$$

Then,
$$\log_{10} M_0 = M_{TSU} + 20$$

• IT WORKS!!



[Okal and Titov, 2006]

RECALL MANTLE MAGNITUDE

[Okal and Talandier, 1989]

$$M_m = X(\omega) + C_D + C_S + C_0$$

- Applied to mantle Rayleigh waves; typically, T = 50 to 300 seconds.
- $X(\omega)$ is spectral amplitude in μ m*s
- *C_D* is distance correction
- C_S is source (frequency) correction
- $C_0 = -0.90$ is locking constant (predicted theoretically)

THEN, M_m is directly related to seismic moment M_0 :

$$M_m = \log_{10} M_0 - 20$$

 M_m combines simple "quick-and-dirty" concept of one-station *magnitude* with modern analytical approach (measuring a *bona fide* physical quantity, the seismic moment, using physical units). It does not saturate.

Valid even for 1960 Chilean earthquake.

A tsunami wave on the high seas is a branch of normal modes of the Earth [Ward, 1980].

 \rightarrow QUESTION: Can we extend the concept of M_m to a tsunami wave measured on the high seas -- and call it M_{TSU} ?

DEVELOPING A FORMULA FOR M_{TSU}

The basic formula for the spectral amplitude of a spheroidal wave by a dislocation remains applicable:

$$X(\omega) = M_0 \cdot a \sqrt{\frac{\pi}{2}} \cdot \frac{1}{\sqrt{\sin \Delta}} e^{-\frac{\omega a \Delta}{2UQ}} \cdot \left[\frac{1}{U} \mid s_R l^{-1/2} K_0 - p_R l^{3/2} K_2 - i q_R l^{1/2} K_1 \mid \right]$$

$$M_m = \log_{10} M_0 - 20 = \log_{10} X(\omega) + C_D + C_S + C_0$$

Need only adjust the corrections C_D and C_S and the constant C_0 .

THE DISTANCE CORRECTION C_D

$$C_D = \frac{1}{2} \log_{10} \sin \Delta$$

THE SOURCE (FREQUENCY) CORRECTION C_S

$$C_S = -\log_{10} \left[\frac{\langle | s_R - p_R | \rangle \omega^{1/2} g^{-3/4}}{8\pi \mu a^{3/2}} \cdot H^{-3/4} \right]$$

$$C_S = 0.087 \,\theta^3 - 0.069 \,\theta^2 + 0.508 \,\theta + 2.299$$

 $(\theta = \log_{10} T - 3.122).$

(The latter formula uses *Okal*'s [2003] asymptotic expressions of the tsunami eigenfunction to compute the various excitation coefficients for a shallow source in the limit $\omega \to 0$).

DEVELOPING A FORMULA FOR M_{TSU}

The basic formula for the spectral amplitude of a spheroidal wave by a dislocation remains applicable:

$$X(\omega) = M_0 \cdot a \sqrt{\frac{\pi}{2}} \cdot \frac{1}{\sqrt{\sin \Delta}} e^{-\frac{\omega a \Delta}{2UQ}} \cdot \left[\frac{1}{U} \mid s_R l^{-1/2} K_0 - p_R l^{3/2} K_2 - i q_R l^{1/2} K_1 \mid \right]$$

$$M_m = \log_{10} M_0 - 20 = \log_{10} X(\omega) + C_D + C_S + C_0$$

Need only adjust the corrections C_D and C_S and the constant C_0 .

THE LOCKING CONSTANT C_0

• If $X(\omega)$ is the spectrum of the wave height at the surface in $\mathbf{cm}^*\mathbf{s}$, then

$$C_0 = 3.10$$

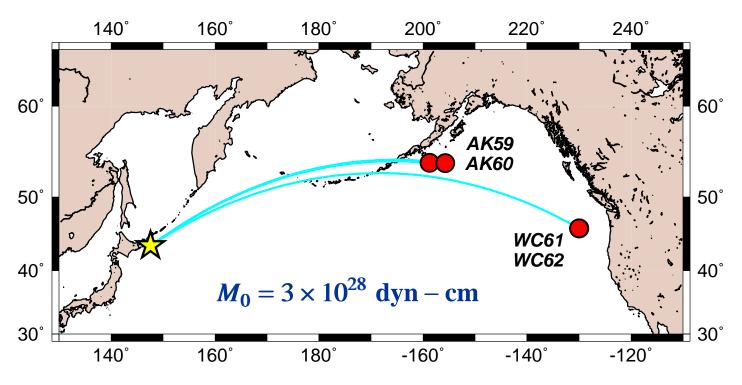
• If one uses the bottom pressure p(t) recorded in $\frac{dyn}{cm^2}$ on the ocean bottom, then use $P(\omega)$ rather than $X(\omega)$; $[P(\omega) = \rho_w g X(\omega)]$, and

$$C_0 = 0.11$$

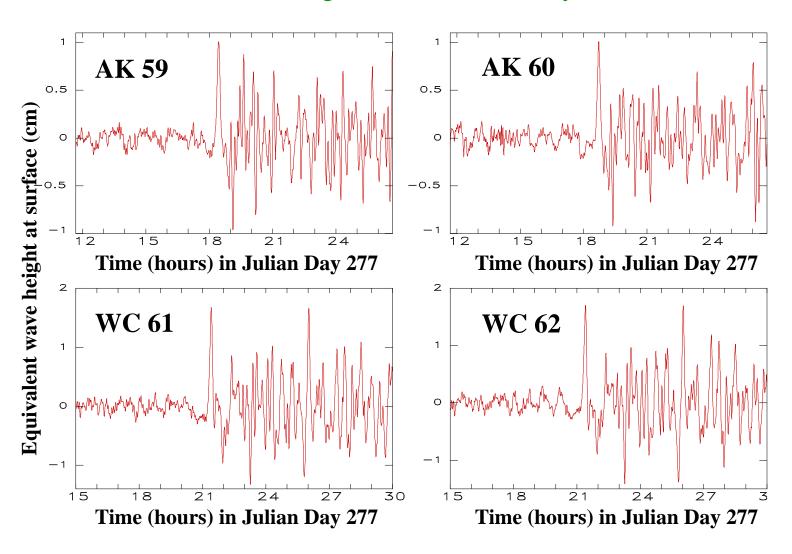
• If p(t) is recorded in **pounds**[-force] per square inch, then

$$C_0 = 4.95$$

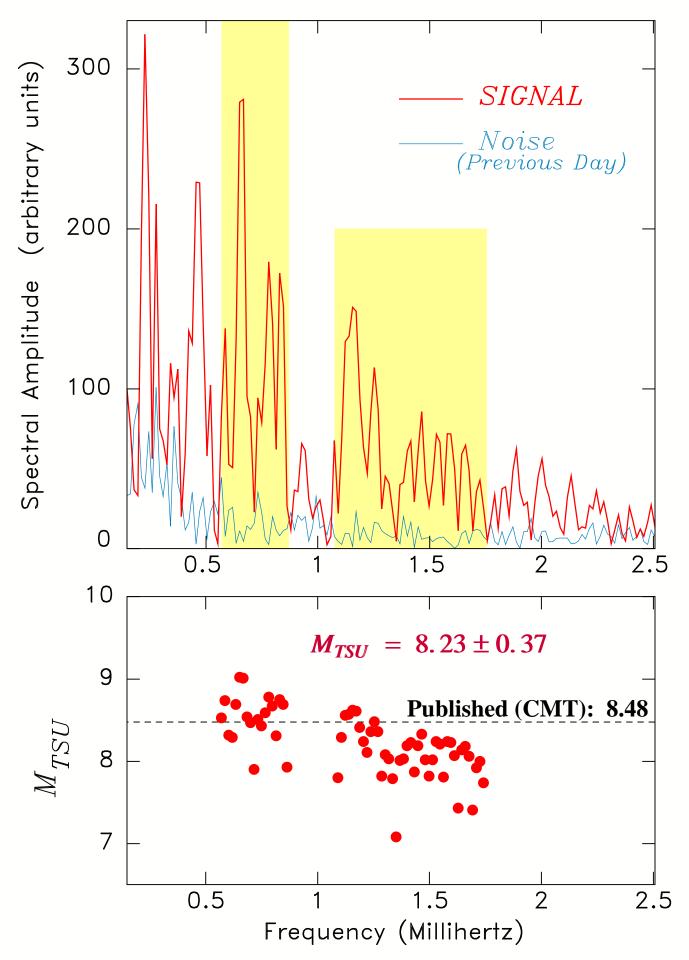
Case Study: KURIL ISLANDS, 04-OCT-1994



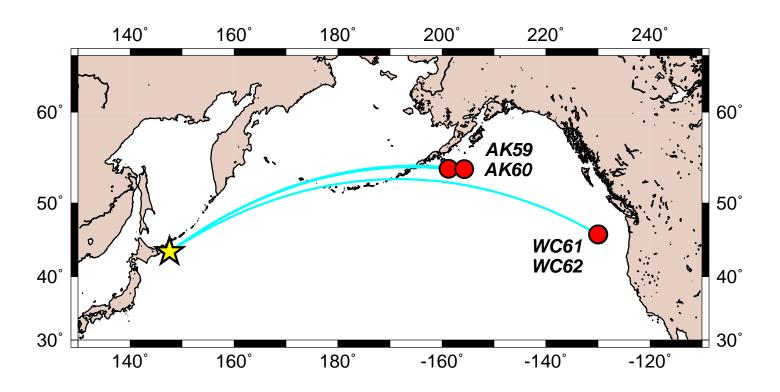
To date, Largest Event Recorded by DART

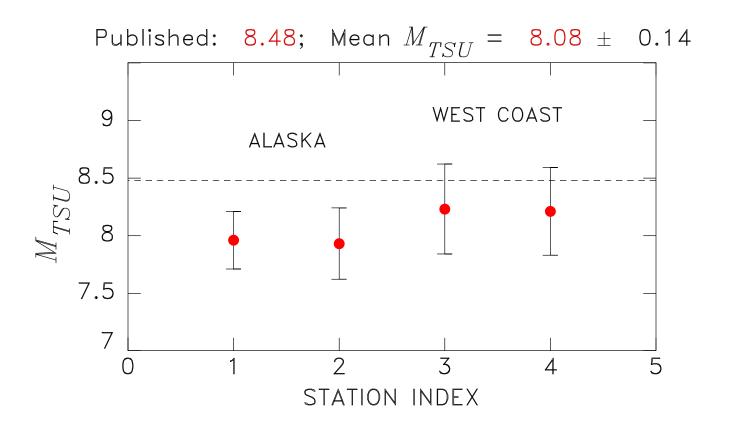


WC62 KURILES 1994

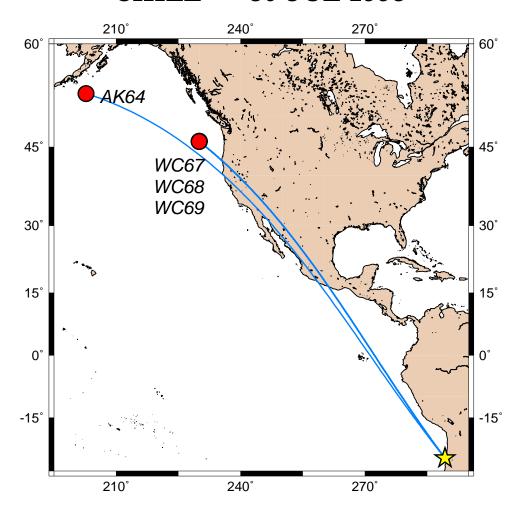


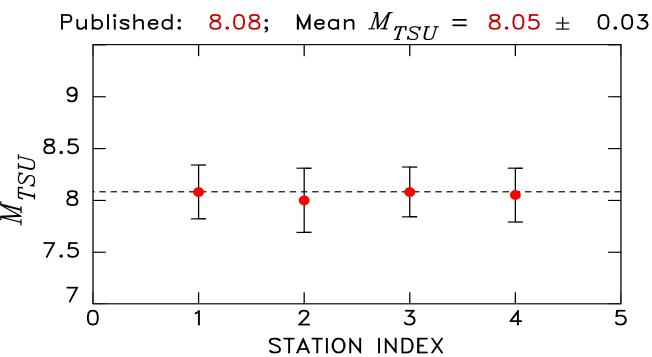
Case Study: KURIL ISLANDS, 04-OCT-1994





CHILE -- 30 JUL 1995





Works despite *UNFAVORABLE GEOMETRY* requiring *NON-GEOMETRICAL propagation* !!

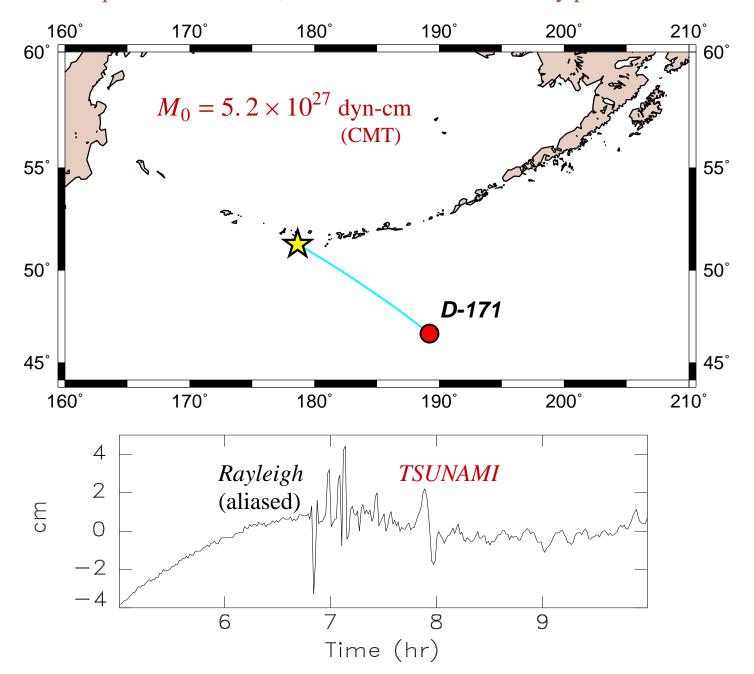
SUCCESSFUL OPERATIONAL USE

17 NOV 2003

This is a smaller earthquake which was not recorded at the Alaskan and West Coast DART gauges.

However, a new station, D-171, is only 900 km from the epicenter, and clearly recorded the tsunami, although at a very coarse sampling (1 minute).

Despite this limitation, the event can be successfully processed.

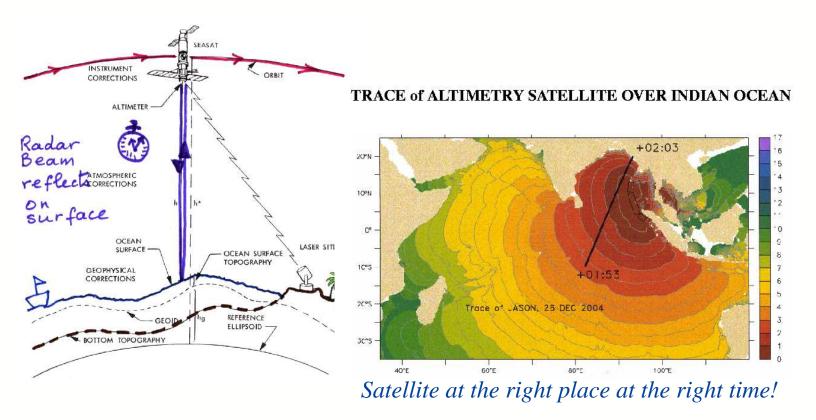


→ This estimate was used in real-time to call off an alert for Hawaii.

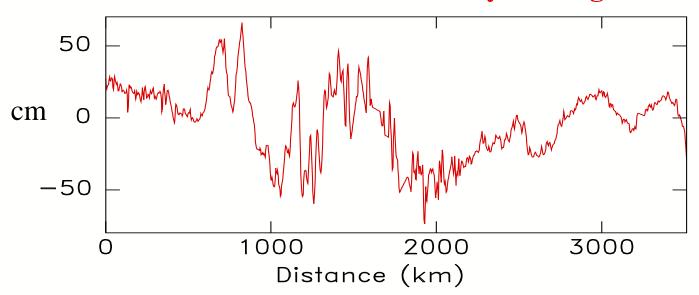
APPLICATION of M_{TSU} to JASON SATELLITE TRACE

DETECTION by SATELLITE ALTIMETRY gives first definitive measurement of *MAJOR* tsunami on *HIGH SEAS*

(previous detection by *Okal et al.* [1999] during 1992 Nicaragua tsunami -- 8 cm -- at the limit of noise).



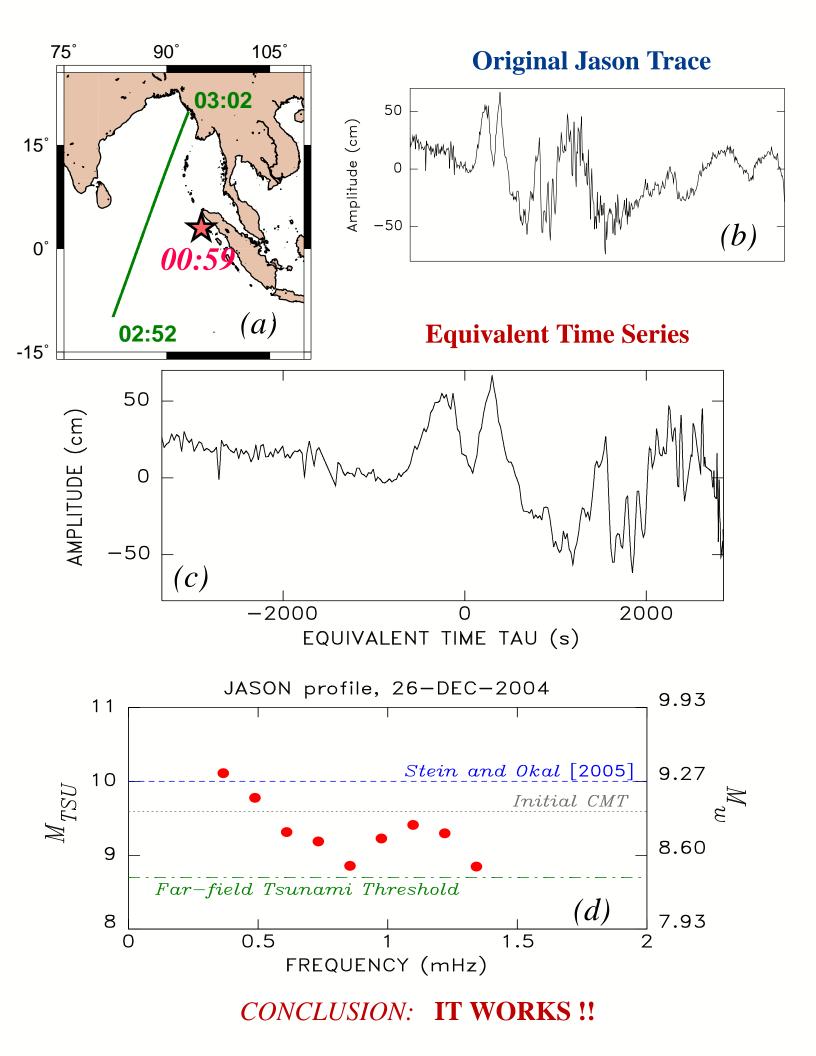
measures 70 cm across Bay of Bengal



• *QUESTION:* Can we quantify the JASON trace, *i.e.*, recover from it the source of the tsunami?

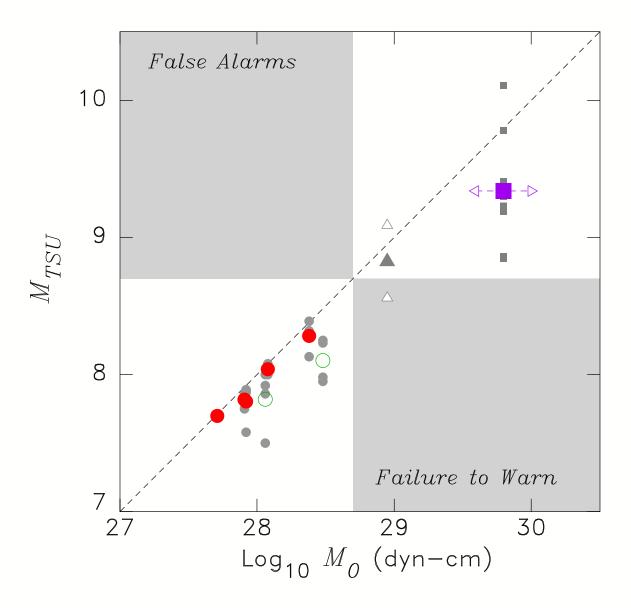
• *PROBLEM*: JASON is neither a time series nor a space series.

• SOLUTION: Rebuild an approximate times series from the JASON trace, then process through M_{TSU} .



$M_{TSU:}$ CONCLUSION

• The algorithm successfully retrieves the seismic moment of the parent earthquake.



• The examples tested suggest that the precision is sufficient to avoid false alarms and failures to warn.